

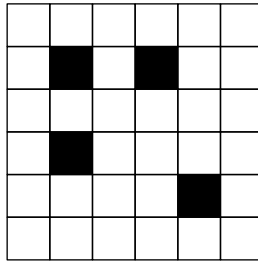
1. A group of students are solving worksheets that consist of math and physics problems. The time required to solve a math problem and a physics problem is different, but each student is able to solve problems of the same subject at the same rate. Pranav is observing the students, and he notices the following:

- It takes 1 hour for 10 students to solve a worksheet consisting of 30 math problems and 20 physics problems.
- It takes 2 hours for 6 students to solve a worksheet consisting of 24 math problems and 30 physics problems.
- It takes 3 hours for 5 students to solve a worksheet consisting of 15 math problems and x physics problems.

Find x .

2. Find the probability that a randomly chosen positive divisor of 15^{225} is an integer multiple of 5^{200} as a fraction in lowest terms.

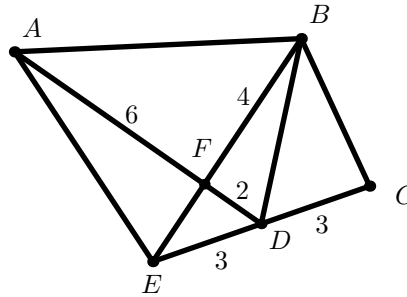
3. Akshar is biking to school from his house. He can only ride to the right and up in the given grid. However, there are 1×1 puddles throughout the grid indicated by black squares, and Akshar cannot travel through any point that encloses a puddle. If his house is located at the bottom-left corner, and the school is located at the top-right corner, how many possible paths are there for Akshar to take from his house to school?



4. Suppose we have a complex number w with real part 225, and a real number n such that $\frac{w}{\bar{w} - n} = 4i$ where \bar{w} is the conjugate of w . Find \sqrt{n} as a common fraction expressed in simplest radical form.

5. Sequence A_n is defined by $A_1 = 1$ and $A_n = A_{n-1} + n$ for integers $n > 1$. Sequence B_n is defined by $B_1 = 1$ and $B_n = A_n + B_{n-1}$ for integers $n > 1$. Find the sum of the exponents in the prime factorization of $B_{2020} - B_{2018}$.

6. In the figure below, points E, D, C are collinear, quadrilateral $ABDE$ is cyclic, meaning all 4 points lie on a circle. $\overline{ED} = \overline{DC} = 3$, $\overline{AF} = 6$, $\overline{FD} = 2$, and $\overline{BF} = 4$. The perimeter of $\triangle BEC$ can be expressed as $a + \frac{\sqrt{b}}{c}$ where a, b, c are positive integers, and b is not divisible by the square of any prime. Find $a + b + c$.



7. Let S be the set of all distinct points in the coordinate plane that form an acute isosceles triangle with the points $(32, 33)$ and $(63, 63)$. Given that a line L crosses S a finite number of times, find the maximum number of times L can cross S .

8. Two lines are drawn, intersecting at angle x . Two circles with radius $r_1 > r_2$ are drawn such that both circles are tangent to these two lines and to each other. If $\sin(x) = \frac{4\sqrt{2}}{9}$, what is the sum of all distinct possible values of $\frac{r_1}{r_2}$? Express your answer in simplest radical form.

9. Pranav has a 10-sided die, and Kenan has two 6-sided dice. What is the probability that the number that Pranav's die shows is larger than the sum of the numbers shown on Kenan's two dice?

10. Let $S_n(x)$ be functions defined on the positive integers x and n as follows:

- $S_1(x) = 2x - 4$

- $S_n(x) = \sum_{k=1}^x S_{n-1}(k)$

Find the sum of the two largest prime factors of $|S_{30}(5)|$.