Power of a Point

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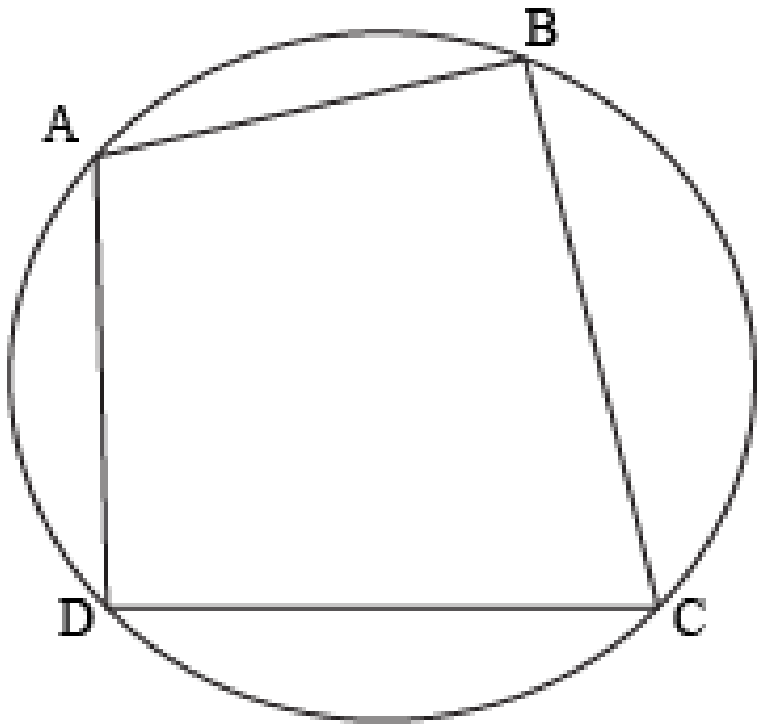
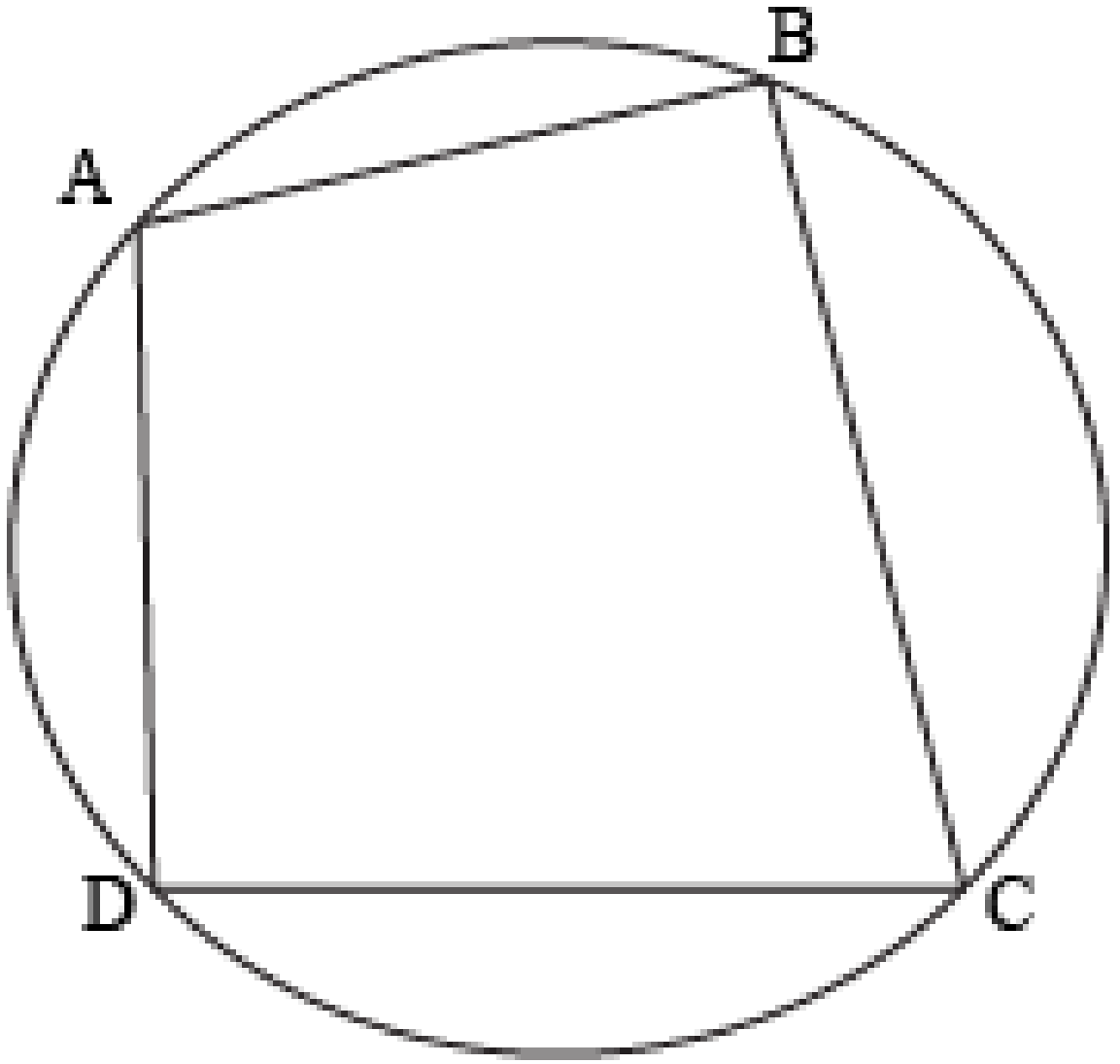
Cyclic Quadrilateral

- A quadrilateral is said to be **cyclic** if its 4 vertices all lie on a common circle.



Conditions for Concyclicity

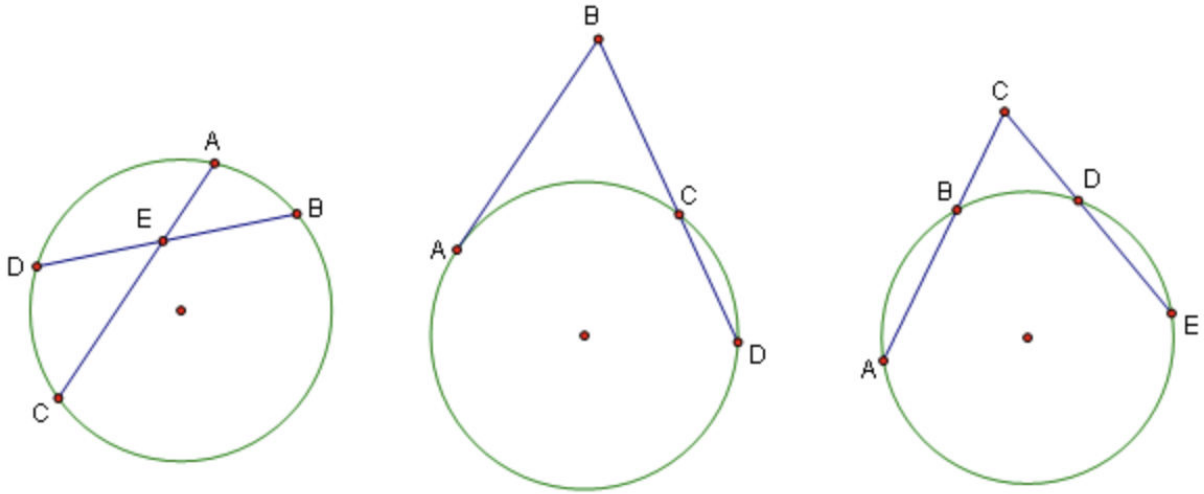
- $\angle A + \angle C = \angle B + \angle D = 180^o$
- $\angle ABD = \angle ACD$
- $\angle BCA = \angle BDA$
- $\angle BAC = \angle BDC$
- $\angle CAD = \angle CBD$



Power of a Point

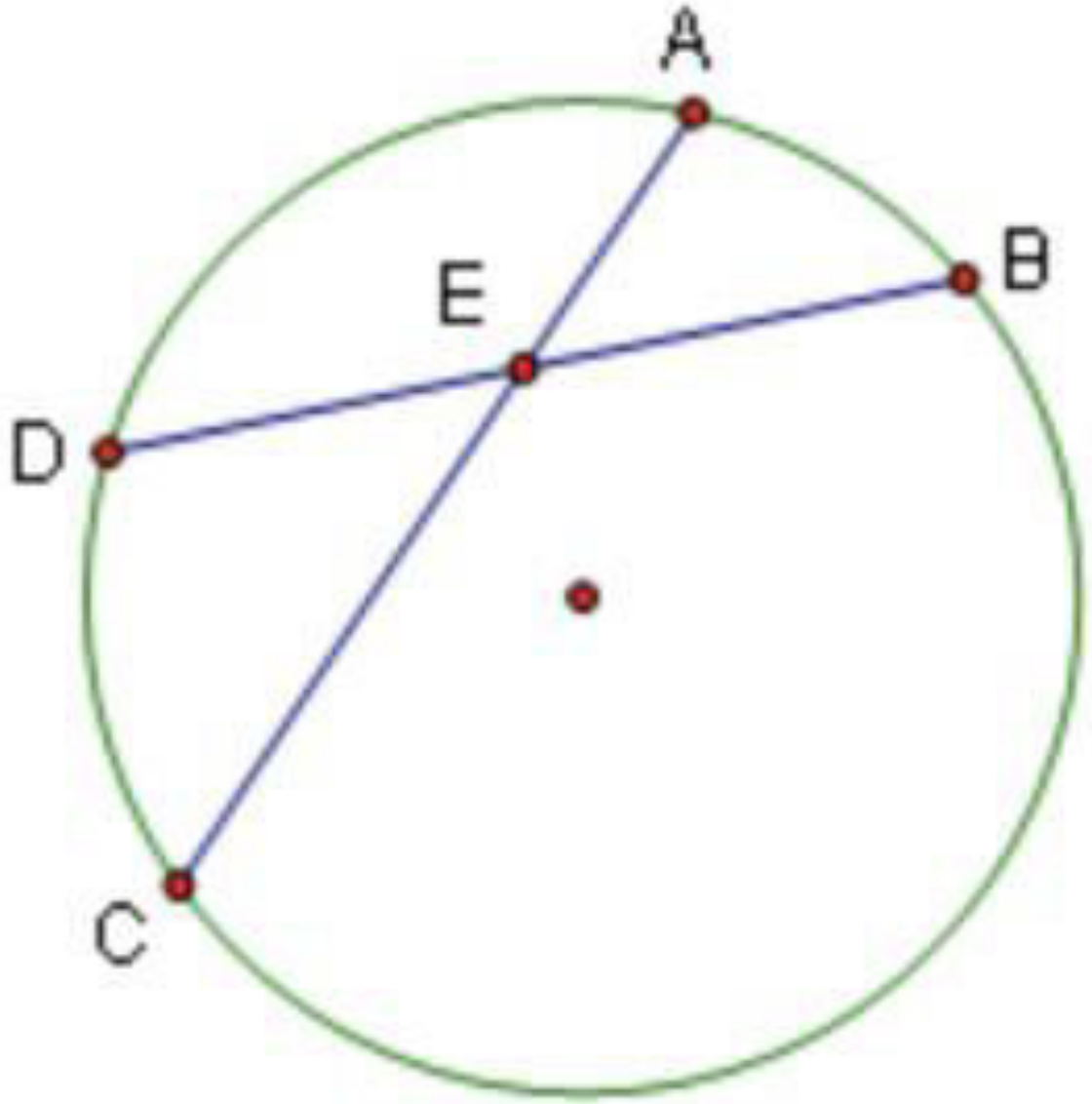
There are three possibilities as displayed in the figures below.

1. The two lines are chords of the circle and intersect inside the circle (figure on the left). In this case, we have $AE \cdot CE = BE \cdot DE$.
2. One of the lines is tangent to the circle while the other is a secant (middle figure). In this case, we have $AB^2 = BC \cdot BD$.
3. Both lines are secants of the circle and intersect outside of it (figure on the right). In this case, we have $CB \cdot CA = CD \cdot CE$.



Proof

$AE \cdot CE = BE \cdot DE.$



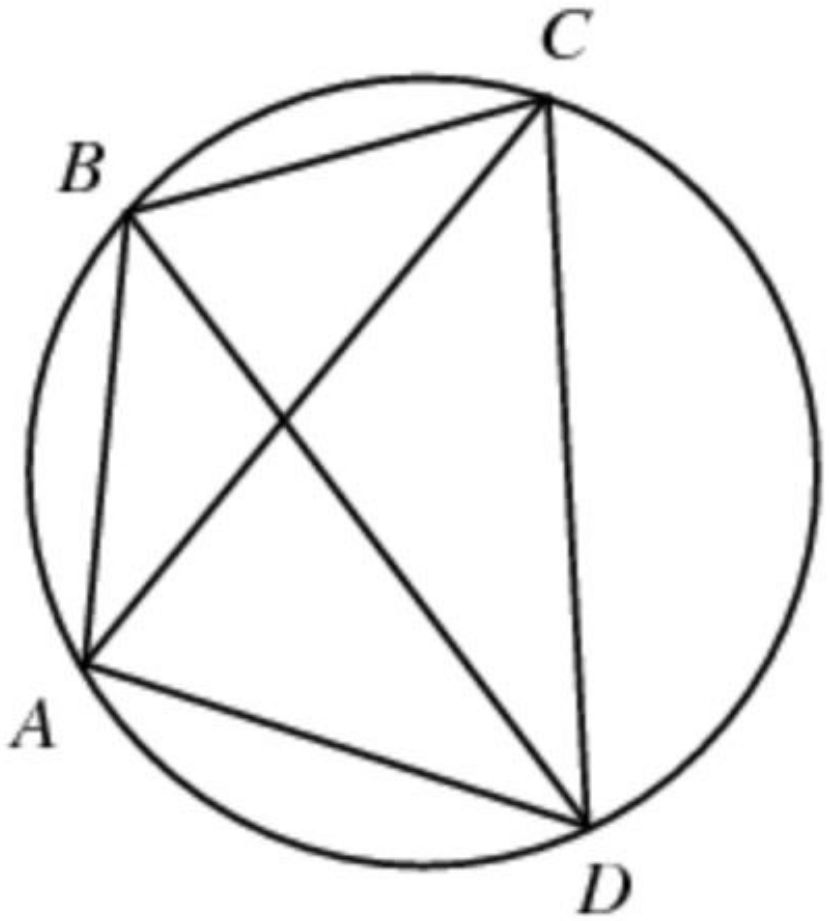
The Power to a Circle

2020 AMC 12B #10

In unit square $ABCD$, the inscribed circle ω intersects $\overline{CD}$ at M, and $\overline{AM}$ intersects ω at a point P different from M. What is AP?

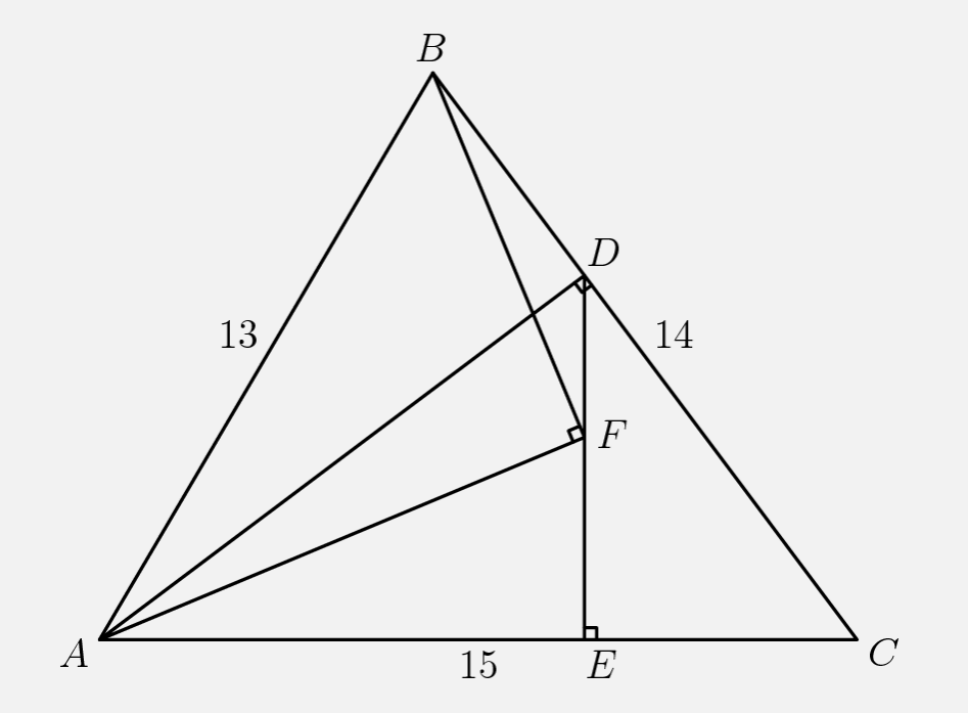
Ptolemy's Theorem

- ABCD is cyclic if and only if $AB \times CD + BC \times DA = AC \times BD$



2013 AMC 10B #23

In triangle ABC, $AB = 13$, $BC = 14$, and $CA = 15$. Distinct points D, E, and F lie on segments $\overline{BC}$, $\overline{CA}$, and $\overline{DE}$, respectively, such that $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AC}$, and $\overline{AF} \perp \overline{BF}$. The length of segment $\overline{DF}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?



2010 USAMO #1

Let $AXYZB$ be a convex pentagon inscribed in a semicircle of diameter AB. Denote by P, Q, R, S the feet of the perpendiculars from Y onto lines AX, BX, AZ, BZ, respectively. Prove that the acute angle formed by lines PQ and RS is half the size of $\angle XOZ$, where O is the midpoint of segment AB.