CNCM Math Bowl Preliminary Round 5

CNCM Administration

Problems

The Warriors and the Cavaliers play in a play-off series, which ends when a team wins 4 games [Each game has a clear winner, no ties]. How many possible ways can the series end in which one team ends up winning 4 games to 2 games?

Answer: 20

The base 8 number $11.\overline{73}$ can be expressed as the base 10 number $\frac{63a+b}{63}$. Find a+b.

Answer: 68

A "split" is defined as expressing the number N as two non-negative integers a and b such that a + b = N. If the number 18 is split into the numbers S_1 and S_2 are split into S_3 , S_4 and S_5 , S_6 respectively, how many possible values of $S_1 + S_2 + S_3 + S_4 + S_5 + S_6$ exist?

An apple pie, cherry pie, and blueberry pie were weighed two at a time at the worlds largest pie contest. The weights were 132 lbs, 98 lbs, and 364 lbs. What is the average of the weights of three pies?

How many ordered pairs of integers satisfy $x^2 + y^2 = 92$?

An equilateral triangle with side length one has vertices A_0, B_0 , and C. Define A_n to be a point on the line $\overline{A_0B_0}$ closer to A_0 than to B_0 that divides the line $\overline{A_{n-1}B_{n-1}}$ in a 1 : 3 ratio, and define B_n to be a point on the line $\overline{A_0B_0}$ closer to B_0 than to A_0 that divides the line $\overline{A_{n-1}B_{n-1}}$ in a 3 : 1 ratio. Define R(n) as the area of the triangle with vertices A_n, B_n , and C, and define P(n) as the perimeter of that same triangle. Extend the domain of R_n and P_n from the non-negative integers to the reals on the elementary non-trigonometric curves that define them on the non-negative integers. Find $\lim_{n\to-\infty} \frac{R_n}{P_n}$.

A die is rolled three times. It is known that a 1 was rolled exactly once. What is the most likely value of the sum of the three numbers on the three faces?

What is (1+i)(1-i)(i+2)(i-2)?

Jevro is selling crayons and pencils, while Andrew attempts to analyze Jevros sales to determine the price of a single pencil. Jevro sells 24 crayons and 44 pencils for 100 Jevro then sells 18 crayons and X pencils for Y dollars. Andrew knows what X and Y are, but it is impossible for him to find out how much a single pencil costs. Find X + Y.

Find the value of

$$\sum_{i=0}^{5} 2^{i} \binom{5}{i} \binom{5-i}{\frac{5-i}{2}}$$

462

What is the largest integer n such that $2^n \ge n^2$? 4

Two trains T_1 and T_2 are going at speeds of 12 mph and 16 mph, respectively. T_2 breaks down and is repaired after x hours. If both trains reach a station 100 miles from the starting point at the same time, find x.

What is the remainder when $2019^{2018^{2017}}$ is divided by 100?

Pipe 1 can fill a tank in 4 hours and Pipe 2 can fill the same tank in 3 hours. How long will it take to fill the tank if both pipes are used?

How many pairs (m, n) of integers satisfy the equation m + n = mn?