CNCM Math Bowl Semifinal 1

CNCM Administration

Problems

Find the product of all the real solutions to $\sqrt{x}\left(x - \frac{4}{9}\right) + 1 - \frac{9}{4}x = 0$.

Sam is shooting 3-pointers at the gym. He has a 30% shooting rate after taking 120 shots. If he makes x shots out of his next $x + 10$ shots, and his shooting rate improved to 41.25%. Find x.

Given that $\dfrac{\sin\left(\frac{\pi}{30}\right)}{\sin\left(\frac{29\pi}{60}\right)} = A\sin(B)$ and $0 < B < \frac{\pi}{2}$, find A and B in radians.

Find $ba^2 + ab^2 + a^2b^2$ where a and b are the roots of equation $2x^2 + 6x + 4 = 0$.

Suppose a is the largest positive integer less than 1000 such that a can be expressed as the difference of 2 consecutive cubes and $2a + 79$ is a perfect square. Let r be the remainder when a is divided by 13. Find the value of r.

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Find the sum of the last 3 digits of 17^9.

Points A, B, C, and D are four distinct points equidistant from point E. AE has length 12 and AC is perpendicular to BD, length $AB = AD$ and $BC = CD$. Given that BC is an integer, what is the largest area that is inside the circle with center E and radius 12 that is outside the quadrilateral? Express your answer in simplest radical form.

A circle and a square have equal area. The ratio of the length of the longest line segment that has both endpoints on the square and the length of the longest line segment that has both endpoints on the circle is denoted as R. What is

$$\cos\left(\sum_{k=1}^{237} kR^2\right)$$

Note that $\cos(A) = \cos(A + 2\pi)$.

Cyclic quadrilateral $ABCD$ has diagonals AC and BD that intersect at point E. Let $AB = 14, AE = 3, BE = 12, CE = 8$. Find the product $(AD)(BC)$.

Answer: $\boxed{\dfrac{70}{3}}$

The equation $x^4 = 13x^2 + 42$ has two real roots. The sum of the roots squared can be expressed as $a + b\sqrt{c}$. Find $a + b + c$.

Find the number of ordered pairs of integers (a, b) where $a, b > 2$ such that $2^a + 1$ is divisible by $2^b - 1$.

Answer: $\boxed{0}$

Suppose we have 3 non negative real numbers x, y, and z. Find all possible values of t such that $x^t(x-y)(x-z) + y^t(y-z)(y-x) + z^t(z-x)(z-y) \geq 0$.

$t \in [0, \infty) thisistheanswer$

Find the largest cube less than 179446 that can be expressed as a difference of squares.

If $x = 3^{\log_3 9}$, find $\log_x 216 + \log_x \dfrac{1}{8}$.

Find the sum of all real numbers x such that $\sqrt[4]{16x^4 - 32x^3 + 24x^2 - 8x + 1} = 5$.

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