CNCM Math Bowl Semifinal 2

CNCM Administration

Problems

A colony of 20 bees can build a beehive in 80 days. However, after 20 days, the bees decide they want the hive to be built in 60 days rather than 80. How many more bees should be taken on to make this possible?

Given that $f(x)$ is a linear function such that $\lfloor 2x + 1 \rfloor + \lceil f(x) \rceil = 35$ for all real x, find $f(x)$.

Find all ordered pair solutions of positive real integers to $x^2(1+y^2)+y^2(1+z^2)+z^2(1+x^2) \geq 6xyz$.

(0,0,0), (1,1,1)

Find the ordered pair of positive integers (m, n) that satisfies $1 + 5 \cdot 2^m = n^2$.

Evaluate the sum of $\dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \cdots + \dfrac{1}{n!} + \ldots$

The expression $39x + 42y$ gives a remainder of 45 when divided by 54 where x and y are integers. Find the remainder when $x + 8y$ is divided by 18.

Two points of a triangle lie on the endpoints of diameter of a circle with radius 2. If S is the area of the triangle that lies inside the circle, find all values that S can take on. Give your answer in interval notation.

A point $P(x_1, x_2)$ is reflected over the line $y = 5$ onto P'. Find the sum of the x and y coordinates of P' in terms of x_1 and x_2 in simplest form.

There are 5 keys, numbered $1 - 5$, that are placed randomly onto 5 slots on a key chain. How many such key chain arrangements are there such that keys 4 and 5 aren't adjacent to each other? Arrangements that are flips and rotations of other arrangements are NOT considered distinct.
Answer: $\boxed{6}$

Sonic and The Flash play a game with dice. Sonic rolls the dice. If he rolls a 1 or 2, he wins. If he rolls a 3 or 4, the Flash wins. If a 5 or 6 is rolled, Sonic rolls again. What is the probability that Sonic wins the game?

Find the value of

$$\sum_{i=0}^{4} \binom{13}{3i} - \sum_{i=0}^{4} \binom{12}{3i}$$

1365

Find the value of

$$\prod_{n=0}^{\infty} \cos\left(\frac{\frac{\pi}{8}}{2^n}\right)$$

$$\frac{\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}{\frac{\pi}{8}}$$

Find the sum of the solutions of the equation $|x+7| + 2 = |x+2| + 5$

What is the least value of N such that $(AB)^N = 1$ for all ordered pairs of complex numbers (A, B) such that $A^{210} = 1$ and $B^{63} = 1$?

A rhombus of side length 4 is such that its internal angles are 60 and 120 degrees. An ellipse is circumscribed about the rhombus such that the vertices of the rhombus lie on the ellipses major and minor axes. Find the area of the ellipse.