CNCM Math Bowl Finals

CNCM Administration

Problems

Define $f(x)$ as the exponential function that outputs the number with x 4s in a row for all integers x; for example, $f(2) = 44$. Let $g(x)$ be the extension of $f(x)$ onto the reals; that is, $g(x) = f(x)$ for integer numbers, but g is defined along the continuous exponential curve of f for non-integers. What is $g\left(\frac{1}{2}\right)$?

Find the ordered pair of positive integers (m, n) such that $2^n + n = m!$.

Let a, b, c, d be the roots of $x^4 + 9x^3 + 9x^2 + 27x + 97$. Find $\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}\right)$. **Answer:** $\boxed{1566}$

The number $N = {p_1}^a \cdot {p_2}^b \cdot {p_3}^c$ for distinct positive primes p_1, p_2, p_3 such that N has 12 divisors. How many ordered triples of positive integers (a, b, c) are there?

How many different arrangements of ABRACAD are there such that no two As are consecutive?

A regular polygon has an external angle measure of $20°$. The polygon has a sides and b diagonals. If $a + bi$ is a root of he monic quadratic $x^2 + cx + d$, find d.

We have $x^3 + y^3 + z^3 = 33$ for three integers x, y, and z such that $x > y > z$. Given that x, y, and z each have the same number of digits and given that x is the only positive number of the three, what is the least number of digits that x can have?

The roots of $2x^2 + \dfrac{2}{x^2} = -1 + \sqrt{5}$ form a quadrilateral in the complex plane. Find the square of the area of this quadrilateral.

There exists unique digits A and B such that 99 divides the number $1A58B3$. Compute $A + B$.

Triangle ABC has $AB = 7020, BC = 2925, AC = 7605$. What is the ratio of the area of the corresponding circumcircle and the corresponding incircle? (Your answer should be greater than one)

Given that A, B, C, and D are chosen randomly from the interval $(-1, 1)$, what is the probability that $A^2 + B^2 + C^2 + D^2$ exceeds 1?

Jackson writes the letters $CNCM$ in order and keeps writing down the same letters until he has written down 1738 letters (ie $CNCMCNCMCNCM\dots$) Let the number of times the string MCN appears in his writing be denoted as E. How many positive factors of E are less than $\sqrt{E}$?

Everybody knows:

- There is a sequence of three positive integers.
- It is in either arithmetic or geometric progression.
- The first term is 128.
- The second term is less than 128.
- Rajking is perfectly rational, and knows the value of the second number in the sequence.

Rajking knows:

- The value of the second number.

Rajking makes an informed guess of the value of the third number in the sequence. What is the probability that Rajking correctly guesses the third number in the sequence from your perspective?

There are 4 circles of radius one shown below. Let S be the set of all triangles which contains all 4 circles. Find the area of the intersection of all triangles in S.

Compute

$$\sum_{k=0}^{10} k^3 - 9k^2 + 26k - 27.$$