

# CNCM Math Bowl Team Round

Central NC Math Group

The CNCM MathBowl Team round is a 20 minute, 10 question exam. You are allowed no external aids, and are only to work with your own team. Do not be discouraged if you cannot solve all of these problems, as they are designed to be difficult to solve by experienced problem solvers.

1. A coloring of a regular pentagon is created by coloring each of its edges red, blue, or green. How many colorings are there such that no two consecutive edges of the pentagon are the same color?

2. A certain ternary sequence is created by letting the first digit be 1, and adding on any combinations of the following:

- A string of three 0s
- A string of two 1s
- A string of two 2s

How many ways are there to create a 14-digit sequence using this method?

3. Suppose we have a polynomial  $P(x)$  such that  $P(x) = (x - 27)^{27} + (x - 27)^{26}(x - 13) + (x - 27)^{25}(x - 13)^2 + \cdots + (x - 27)^{15}(x - 13)^{12} + (x - 27)^{14}(x - 13)^{13}$ . Find the sum of all values  $x$  such that  $P(x) = 0$ .

4. Consider the function  $y = x!$ . Define  $f(x)$  as the line with endpoints  $(a, a!)$  and  $(a + 1, (a + 1)!)$  for every pair of aforementioned points. Find the intersection of  $y = f(x)$  and  $y = 60x + 45$ .

5. How many squares can be formed such that each vertex of each square lies on a lattice point of a  $6 \times 5$  grid? Note that two squares are considered distinct if they do not share all four vertices.

6. How many points  $(x, y)$  are there that satisfy the following?

- $x$  and  $y$  are integers
- $x < 25, y < 25$
- $(x, y)$  forms an isosceles triangle with  $(-5, 0)$  and  $(5, 0)$  in the  $xy$  plane.

7. The graph of  $y^2 = (x - 1)^{(x-2)}$  is shifted down five units and right two units. The graph of the new equation can be written as  $(y + k)^2 = (x + a)^{(x+b)}$  where  $k, a$ , and  $b$  are (not necessarily positive) real numbers. Find  $k + a + b$ .

8. Find

$$\lim_{N \rightarrow \infty} \left( N \sin \left( \frac{\pi - \frac{2\pi}{N}}{2} \right) \sqrt{2 - 2 \cos \left( \frac{2\pi}{N} \right)} \right)$$

**9.** The zebra is a made-up chess piece that moves three spaces in one direction and then 2 spaces in an orthogonal direction. The zebra starts at the corner of an  $8 \times 8$  chess board and begins to make random, legal moves. How many moves is it expected to make before it returns to the corner it starts at? Include the final move that lands the zebra at its original corner in your answer.

**10.** In each round of the game Vorten, Rahul and Steve each simultaneously write down a letter down on a blank piece of paper hidden from their opponent:  $T, M, S,$  or  $A$ . They then simultaneously reveal their letters. The table below shows their respective payoffs for each pair of decisions. Numbers listed are the number of dollars Steve pays Rahul per decision set, and negative numbers indicate Rahul pays Steve. Given that Steve cannot take advantage of Rahul's strategy, determine Rahul's strategy. Write your answer in the form  $(P_T, P_M, P_S, P_A)$  where the variables  $P_N$  represent the probability with which Rahul writes down letter  $N$ .

0	T	M	S	A
T	0	-200	200	200
M	200	0	200	-300
S	-200	-200	0	200
A	-200	300	-200	0