CNCM MathBowl Individual Round

CNCM Administration

May 18th, 2019

The individual round examination of the CNCM MathBowl is a 10 question, 50 minute test. You are allowed no external aids, including a Calculator, Phone, Smartwatch, or any other WiFi capable device. Write your answers in the box provided, for only answers inside the boxes will be graded. There is no partial credit. Your score is calculated as the number of questions you got correct. There is no benefit from a blank answer and no deduction from a wrong answer. All answers must be expressed in exact form unless otherwise specified.

1. Kevin is playing Clash of Guilds. He has 300 housing spaces available. 1 dragon uses up 40 housing spaces and 1 math trooper uses up 20 housing spaces. How many different armies of dragons and math troopers are possible? (Kevin has to use all of his housing spaces.)

2. Define A such that

- $A_0 = -37$
- $A_n = \frac{A_{n-1}}{2}$ if $A_{n-1} \geq 2$
- $A_n = A_{n-1}$ if $A_{n-1} \leq 2$

Find A_{2020}.

3. Consider the following system of equations.

- $a + b + c = 6$
- $a^2 + b^2 + c^2 = 26$
- $a^3 + b^3 + c^3 = 126$

Given that $a \geq b \geq c$, find the ordered pair (a, b, c).

4. There is a string of length n with randomly generated digits from 0 to 9. What is the smallest value n can be such that the probability that at least two of the digits are the same is at least 50%?

5. 30 teams are competing in the MCNC BowlMath competition, in which every team plays every other team exactly once. Each team has a 50% chance of winning any arbitrary match they play in. The probability that two teams win the same number of games if there are no ties throughout the competition can be expressed as $\frac{a!}{2^b}$. Find $a + b$.

6. Suppose we have $f(z) = a + bi$ for all positive integers a and b. f has the property that the image of each point in the complex plane is equidistant from both that point and the origin. Given that $|a + bi| = p$ and $b^2 = \frac{m}{n}$, where m and n are positive integers, find the value of p such that $m + n = 327$.

7. Find the area of the graph $|x-1|+|x-2|+|x-3|+|x-4| < 14$. that lies below $|x-1|+|x-2|+|x-3|+|x-4|$ and in the first quadrant.

8. An inversion is a function that maps a point a to a not necessarily different point b (both in $\mathbb{R}^2$) such that

- $\overline{AO} \times \overline{BO} = 1$
- A, B, and O are collinear
- O is never in the middle of A and B
- O is the origin, $(0, 0)$

Inverting a geometric object is equivalent to inverting each individual point that composes the object. What is the area enclosed by the shape resulting from inverting a sqaure of side length one centered at the origin?

9. Three regular 7-sided dice, two regular 5-sided dice, and one regular 4-sided die are rolled. The probability that the 6 dice sum to a number divisible by 3 can be expressed as $\frac{m}{n}$, where m and n are positive integers. Find m.

10. Let $\triangle ABC$ be a triangle where $\overline{AB} = c$, $\overline{BC} = a$, and $\overline{AC} = b$. Let point Z be a point on $\overline{BC}$ such that if points D and E are the intersections of $\overline{AZ}$ with the common external tangent lines of w_1 and w_2, where w_1 is the circumcircle of $\triangle ZAB$ and w_2 is the circumcircle of $\triangle ZAC$, then

$$\frac{\overline{AZ}^2}{\overline{DC}^2} + \frac{\overline{BC} \times \overline{CZ}}{bc} = 1$$

Find all possible lengths of $\overline{BZ}$ in terms of a, b, and c.